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Abstract

The question is raised whether there are objective criteria for differentiating individual income, especially between different professional groups.

Criteria are developed for an optimal employment structure. The optimal employment structure is calculated for a test example with the help of the simplex algorithm.

A dynamic mathematical model of a labor market is developed and simulated by a computer program. It turns out that there is an optimal wage structure which means that every worker achieves the highest personal income exactly where the worker would have to work according to the optimal employment structure, depending on the worker's skills.

Within a market economy, individual work performance can be assessed and thus differentiated by objective criteria. However, unlike physical performance, its value is not a fixed value. A certain work performance is to be assessed differently compared to other work performances in different societal needs structures. This should be done by objective criteria.

The Marxian concept of the societal necessary working time can be quantified with these models.

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1 Introduction

The capitalist market economy claims that it is a strictly performance-oriented society. For the wage-dependent worker this should mean that whoever performs on a high level in terms of quantity and quality receives a relatively high share of the total societal product through the payment of a relatively high wage for individual use. For the entrepreneur who generates income through the utilization of the capital this means that a manager who excels in competition due to highly skilled business activity generates a high profit. In addition to this “positive” performance principle, there is also the “negative” performance principle. For the workers that means that especially the "underperforming" workers are always threatened by the loss of their job. For entrepreneurs that means that the “weaker performers” are also threatened by bankruptcy. Both creates a considerable pressure to perform on each individual, so that within the capitalist market economy this performance principle is seen as a sufficient stimulation of individuals and is thus used as an almost exclusive principle of motivating individuals.

For whatever reasons there are huge differences in income in the capitalist market economy, which, because of their apparent spontaneity, are often to be accepted as an inherent characteristic of the (capitalist) market economy. Nevertheless, the question remains whether this capitalist performance principle contributes to the creation of optimal economic structures in the interests of society as a whole and thus to create economically and politically stable conditions in society.

The socialist planned economy also claimed to offer reward according to the performance principle. However, it deliberately refrained from applying the “negative” performance principle. As a substitute for this, attempts were made to use more idealistic performance stimulation in the form of “socialist competition”, which essentially failed. The wage structure did not develop in a spontaneous process of supply and demand. It was also not negotiated in a combative dispute between the organized representatives of two classes, but rather determined administratively. Since the principle of the greatest possible social equality was sought for ideological reasons the wage differences under socialism were relatively small.

In discussions about the performance principle it is usually fairly easy to reach a consensus that the income differentiation in socialism was too small to produce an effective performance principle. And in capitalism it is too big to achieve a sustainable, socially acceptable capitalism. I have not yet heard any plausible considerations for where there should be an objectively justifiable, fair differentiation in between.

A premise for answering this question is the clarification of the concept of “performance” for human work within the societal reproduction process. The use of the terms “performance” and “work” in physics serve as inspiration. In physics a scalar value can be specified for the amount of work performed in a unit of time, regardless of the physical form in which it is performed. Since all forms of energy can actually be converted into one another, there are reproducible conversion factors for various performances, regardless of whether these are mechanical, electrical or thermal performances, for example.

For human labor, too, there are some professions where it seems to be possible and to make sense to quantify the work. These are manual activities that are constantly repeated in the same form with negligible differences in quality. Here the performance can be viewed as proportional to the number of items produced (piece wage). But that is about it. Already trying to compare different manual tasks with one another is difficult. For example, how do you compare the performance of cutting hair with the performance of milking a cow? Even within a single professional group it is often difficult to evaluate individual performance. For example, who can objectively evaluate the differences in performance between two teachers?

This results in the question whether it makes sense at all to introduce the concept of “performance” for human work, which suggests that this performance is also measurable. Unfortunately, despite the difficulties, there is no way to simply avoid the problem. It is necessary to install a distribution system of goods in a societal system in which people work together and consume individually, because the individual must be motivated to participate in the joint work, which requires effort to use its abilities as best as possible. It is of course tempting to activate the quantitatively and qualitatively different abilities of the members of society through a differentiated distribution of goods, which was only possible through the regular production of a surplus product.

Marx assumes the measurability of human labor by regarding the necessary working time as the measure of the work done. He writes: *„But how does one measure quantities of labour? By the time* *the labour lasts, in measuring the labour by the hour, the day, etc“* . In doing so, he takes into account that there are individual differences in the quantity and quality of the various professions as he continues writing: *„Of course to apply this measure, all sorts of labour are reduced to average or simple labour as their unit."* **[2]** Elsewhere he also comments on this: *„Andrerseits muß in jedem Wertbildungsprozeß die höhere Arbeit stets auf gesellschaftliche Durchschnittsarbeit reduziert werden, z.B. ein Tag höherer Arbeit auf x Tage einfacher Arbeit. Man erspart also eine überflüssige Operation und vereinfacht die Analyse durch die Annahme, daß der vom Kapital verwandte Arbeiter einfache gesellschaftliche Durchschnittsarbeit verrichtet." (On the other hand in each averaging process, the higher work always has to be reduced to average work, e.g. one day of higher work to x days of simple work. This saves a superfluous operation and simplifies the analysis by assuming that the worker employed by capital does simple average societal work.)* **[3]** He does not go into any further detail, so that the problem of evaluating individual work could of course not be adequately clarified by him. At that time other problems had priority, which is the formulation of the law of value.

The protagonists of the capitalist market economy leave this problem, like so many others, to their universal control mechanism "supply and demand". They save themselves intensive thinking about it according to the motto "The market will fix it".

In my market economy models **[4]** I also initially avoided the problem by idealizing the workers into an economic subject with a uniform, universally applicable labor. In doing so, I tacitly assumed that every worker who finds work uses its labor with normal intensity, without worrying about what motivates the individual.

Since the functioning of a market economy cannot be made plausible without the formulation of the facts that motivate the individual to act I will now focus on this problem in more detail. I will not base it on a mechanistic concept of performance but approach the problem from a different angle to obtain a more nuanced view of the problem and facilitate objectively quantifiable statements.

2 Description of the problem

What is the function of a differentiated remuneration system related to a free labor market with collective production?

First, it should ensure that a worker who engages in a profession makes the most of its skills in that profession. That results in a higher wage for higher use in a profession for which use is quantifiable at least in comparison to similar activities. The size of the benefit can also refer to one or more indicators o the quality of the work. However, the wage does not necessarily have to be proportional to the quantity of the use. This still corresponds to the traditional understanding of the performance principle.

Secondly, society as a jointly producing association of people is interested in the members of society being distributed among the professions according to their different physical and mental abilities and according to the different requirements of the various professions, so that a maximum and at the same time needs-based assortment of work is given. This range of performances is reflected in a maximal and needs-based range of products and services.

Expecting a remuneration system in connection with a free labor market to solve this complex task cannot be taken for granted. But it is worth the effort to investigate this in detail. Because only under these conditions a free labor market does make sense, where everyone only has to strive for employment in the profession, where everyone can count on the highest possible income for themselves according to their abilities. A labor market that works like that would save a lot of administration. Even if it can be assumed that not everyone can correctly assess their own personal skills, it can be assumed that in an honest effort to choose the right profession for oneself the macrosocial hit rate is likely to be higher than in any administrative assessment of the skills of the individual members of society.

A science-based answer to this question can also be expected to answer the question of the necessary amount of differentiation in wages. This in turn could answer the question of whether it is possible to give a market economy the attribute “social”.

In order to theoretically answer this question a mathematical model of a labor market with a corresponding wage system shall be developed below.

3 Mathematical description of an employment structure

Primary parameters:

- There are **a** workers.
- These workers are employed in **m3** different professions, which are numbered with the index **j3**.
- It is assumed that the performance within a profession can be measured. Accordingly there can be a special unit of measurement or a **normal $\mathbf{Md_{j3}}$ for work performance** for each profession.
- A worker **j1** has a certain individual skill for each profession j3 to achieve a corresponding productivity $\mathbf{P_{j1,j3}}$ with normal effort. The totality of all productivities $P_{j1,j3}$ of a worker j1 results in a vector $\mathbf{P_{j1}}$, which we call productivity profile. Usually every worker has its own individual productivity profile. To reduce the math we assume that there can be several workers with the same productivity profile. These workers are combined to form one productivity group.
- Accordingly, the assumption is: There are **m2** different **productivity groups**, which are numbered with the index **j2**. The productivity profile of group j2 is given by the productivity profile vector P_{j2}, which consists of the components $P_{j2,j3}$. The vectors P_{j2} of the productivity groups can be combined into a matrix, which we call the matrix of productivity profiles or briefly the productivity matrix P of society.
- The components $P_{j2,j3}$ of the productivity matrix P are made dimensionless with the corresponding normal_{j3}, so that there is the **dimensionless productivity matrix p** with the dimensionless components $\mathbf{p_{j2,j3}}$.

$$p = \begin{bmatrix} p_{1,1} & \cdots & p_{1,\,j3} & \cdots & p_{1,\,m3} \\ \vdots & & \vdots & & \vdots \\ p_{j2,1} & \cdots & p_{j2,\,j3} & \cdots & p_{j2,\,m3} \\ \vdots & & \vdots & & \vdots \\ p_{m2,1} & \cdots & p_{m2,\,j3} & \cdots & p_{m2,\,m3} \end{bmatrix} \qquad \text{with} \quad p_{j2,j3} \underset{def}{=} \frac{P_{j2,j3}}{Md_{j3}} \tag{1}$$

- Each productivity group j2 includes $\mathbf{ap_{j2}}$ workers who have the corresponding productivity profile. All these numbers can be summarized in a vector ap, which consists of the **m2** components $\mathbf{ap_{j2}}$ and which we call the **vector of the productivity group sizes**.

In fact, every worker has its own productivity profile. By introducing the productivity groups we have not made any significant restriction to the general case because if we set all productivity group sizes equal to one then again an individual productivity profile is given for each worker.

- When assuming full employment the sum of all productivity group sizes is equal to the number **a** of workers.

$$a = \sum_{j2=1}^{m2} ap_{j2} \tag{2}$$

- The workers in a productivity group can be employed in different professions. For example, in profession j3 from productivity group j2 the number of $\mathbf{ac_{j2,j3}}$ workers can be employed. This statement can be made for any productivity group and any profession. All numbers $ac_{j2,j3}$ for j2=1 to m2 and j3=1 to m3 result in a matrix ac with m2 rows and m3 columns which shall be called the **employment matrix**.

$$ac = \begin{bmatrix} ac_{1,1} & \cdots & ac_{1,\,j3} & \cdots & ac_{1,\,m3} \\ \vdots & & \vdots & & \vdots \\ ac_{j2,1} & \cdots & ac_{j2,\,j3} & \cdots & ac_{j2,\,m3} \\ \vdots & & \vdots & & \vdots \\ ac_{m2,1} & \cdots & ac_{m2,\,j3} & \cdots & ac_{m2,\,m3} \end{bmatrix}$$

- In the case of full employment applies

$$a = \sum_{j2=1}^{m2} \sum_{j3=1}^{m3} ac_{j2,\,j3} \tag{3}$$

Now another vector shall be introduced as a test criterion for the needs-conformity of the employment structure.

- Let us assume that there is a needs-based national economy in which every company can indicate the volume of employment it needs in each profession in order to provide its production. In order to supply this needs-based producing economy with needs-based and profession-specific work performance a needs-based performance structure can be specified by a **vector db0 of a needs-based performance structure** which components $\mathbf{db0_{j3}}$ indicate for each profession j3 which profession-specific performance according to the profession's requirement must be provided.

$$db0 = \begin{bmatrix} db0_1 \\ \vdots \\ db0_{j3} \\ \vdots \\ db0_{m3} \end{bmatrix}$$

For the sake of simplicity in case of possible economic growth it is assumed that the proportions of the need for profession-specific performances will not change at short notice. This means that every multiple of this vector also results in a needs-based performance structure.

Thus all primary parameters for describing an employment structure have been introduced.

Secondary parameters:

All secondary parameters can be derived from the primary ones. They serve to make it easier to evaluate the later results of an optimization.

For each profession j3 the **normalized productivity** $\mathbf{pn_{j3}}$ is defined by

$$pn_{j3\ def}= \frac{1}{a} \cdot \sum_{j2=1}^{m2} p_{j2,j3} \cdot ap_{j2} \tag{4}$$

This value can be interpreted as the mean productivity of all workers, including those who do not work in this profession. These values would also result in the mean productivity of all workers employed in the profession if the number of workers is very large and the distribution of workers among the respective professions were completely left to chance and their respective skills are not taken into account at all. Thus pn_{j3} is also the **profession-independent mean productivity** of the workers in the entire society. This can be seen as a constant value over time and should serve as a base value to show how actual productivity can be increased by optimizing the employment matrix.

The m3 parameters pn_{j3} are combined to a vector pn, which represents the **normalized, profession-independent productivity profile** of society.

Using the employment matrix ac and the productivity matrix p, for each profession can be calculated j3 which **work performance** $\mathbf{d_{j3}}$ is currently achieved in this profession.

$$d_{j3} = \sum_{j2=1}^{m2} p_{j2,j3} \cdot ac_{j2,j3} \tag{5}$$

The vector **d**, consisting of the m3 components $\mathbf{d_{j3}}$, results in the **structure of the work performance** that is currently done due to the current employment matrix ac.

The current numbers $\mathbf{ae_{j3}}$ of workers who are employed in the respective profession j3 are calculated from the employment matrix ac by

$$ae_{j3} = \sum_{j2=1}^{m2} ac_{j2,j3} \tag{6}$$

They are combined in the vector ae.

For each profession the **mean productivity** $\mathbf{pm_{j3}}$ is defined j3 by

$$pm_{j3\ def}= \frac{d_{j3}}{ae_{j3}} = \frac{\sum_{j2=1}^{m2} p_{j2,j3} \cdot ac_{j2,j3}}{\sum_{j2=1}^{m2} ac_{j2,j3}} \tag{7}$$

Thus pm_{j3} is the mean productivity of the workers actually employed in this profession, which means it is profession-dependent. (Compare indicative value pm with indicative value pn)

The m3 parameters pm_{j3} are combined to form a vector **pm** which represents the **mean profession-dependent productivity profile** of society. With this vector the “societally necessary working time” for a product is given in the Marxian sense.

The ratio between the mean profession-dependent productivity pm_{j3} and the normalized (mean profession-independent) productivity pn_{j3} results in the **factor fp_{j3} for the increase of productivity** in the respective profession j3.

$$fp_{j3} \ {}_{def}= pm_{j3} / pn_{j3} \tag{8}$$

As not every profession requires workers from all productivity groups a reasonable employment policy can ensure that the most productive people work within each professional field. The factors fp_{j3} provide information on the extent to which productivity increases in society are achieved just by employment policy, and thus represent quality criteria of employment policy.

The vector db0 is initially given arbitrarily with regard to the level of its components, since at the moment only the relationships of the components to one another are important for the description of a needs-based performance structure. This vector is normalized by introducing a vector **dbn** for a **normalized,** needs-based **performance structure**.

$$dbn_{j3\ def}= db0_{j3} \Big/ \sum_{j=1}^{m3} \frac{db0_j}{pn_j} \qquad \text{for j3= 1 until m3} \tag{9}$$

Since the value of the denominator in this formula is constant in relation to the index j3, it is obvious that the vector dbn represents a multiple (or a fraction) of the vector db0 and is therefore also a needs-based performance structure. In addition, the vector dbn corresponds to the performance structure of a needs-based producing society that consists only of one “normal” worker, whose individual productivity profile is pn.

The vector d of the current societal performance structure due to the employment matrix ac does not have to be needs-conform. **The largest vector** it contains **which represents a needs-based performance structure** is referred to as **db**. db is then the largest vector of a needs-based performance-oriented structure that is contained in vector d, if for all $\mathbf{db_{j3}}$ with j3= 1 to m3 the following applies:

$$db_{j3} = fg \cdot a \cdot dbn_{j3} \tag{10}$$

where for all applies $db_{j3} \leq d_{j3}$ and for at least one j3 applies $db_{j3}=d_{j3}$.

Hereby the factor **fg** was introduced as well. It represents the **factor of increased overall societal performance** due to the employment matrix ac. It is the most important parameter for evaluating the quality of the employment matrix. Therefore it is used as an objective function of an optimization calculation later on.

The factors $\mathbf{fb_{j3}}$ for evaluating the needs-conformity of the work performances can be defined by

$$fb_{j3\ def}= db_{j3} / d_{j3} \tag{11}$$

Accordingly, for each profession they indicate how large the share of needs-based work is in relation to the total work performed in the respective profession. The aim for all fb_{j3} is to have the value one, because then work is carried out completely needs-based ably.

From these factors the more complex factor fbg can be specified. It provides a quick overview of the needs-conformity of overall societal work performance.

$$fbg \ {}_{def}= \frac{1}{a} \cdot \sum_{j3=1}^{m3} fb_{j3} \cdot ae_{j3} \tag{12}$$

If fbg=1, then in total work is done according to needs. The factor indicates what proportion of workers is involved in the overall societal needs-conform work performance structure db within the possibly non-needs-conform work performance structure d.

4 Demonstration example

The previous remarks will now be illustrated using a small demonstration example. It should be noted, however, that the values given were determined arbitrarily in terms of their selection and size. A concordance with the actual conditions was not aspired, because it is not possible with such a small number of selected professions. It's about the principle.

There are 100 employable members of society (workers), i.e. a=100.

There are 4 different professions, i.e. m3=4

The work performances of the professions can be measured in the corresponding units of measurement:

j3	Profession	Unit of measurement / normal work performance
1	Farmer	100 kg of grain per day
2	Baker	50 kg of bread per day
3	Brick mason	1 m^2 floor space of a building per day
4	Mechanics	0.1 piece of a machine per day

Thus the vector of the units of measurement or of the normals of work performances is given by

$$Md = \begin{bmatrix} Md_1 \\ Md_2 \\ Md_3 \\ Md_4 \end{bmatrix} = \begin{bmatrix} 100 \text{ kg} & \text{grain/day} \\ 50 \text{ kg} & \text{bread/day} \\ 1 \text{ m}^2 & \text{floorspace of a bilding/day} \\ 0.1 \text{ piece} & \text{machines/day} \end{bmatrix}$$

With these normals, all parameters that represent work performances are made dimensionless. Abbreviations with a capital initial letter stand for parameters with dimensions and abbreviations with a small initial letter stand for the corresponding dimensionless parameters.

There are 10 different productivity groups, i.e. m2=10. Each productivity group consists of 10 workers, i.e. apj2=10 for all j2= 1 to m2. The individual productivity profiles of all workers in society are described by the dimensionless productivity matrix p.

$$p = \begin{array}{c} \text{j3:}\ \ 1 \quad\ 2 \quad\ 3 \quad\ 4 \qquad\qquad \\ \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 2 & 3 \\ 3 & 2 & 2 & 1{,}5 \\ 2 & 4 & 3 & 2{,}5 \\ 1 & 1 & 1 & 0 \\ 8 & 2 & 2 & 2 \\ 2{,}5 & 3 & 2 & 1 \\ 6 & 6 & 1 & 1 \\ 0 & 0 & 0 & 6 \\ 1 & 1 & 1{,}5 & 2 \end{bmatrix} \end{array} \begin{array}{c} \text{j2} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \qquad ap = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$

For example the first row of the productivity matrix p signifies that a worker in the first productivity group is able to produce **1**·100 kg of grain per day as a farmer (j3=1), or as a baker (j3=2) to bake **1**·50 kg of bread per day, or as a brick mason (j3=3) to build a part of a building with an average of **2**·1 m^2 of usable floor space, or as a mechanic (j3=4) to build **4**·0.1 machines.

The vector db0 of a needs-based performance structure is given by

$$db0 = [\,2 \quad 2 \quad 3 \quad \mathbf{4}\,]$$

This means that the work performance of **2**·100 kg of grain per day, **2**·50 kg of bread per day, **3**·1 m^2 of floor space of a building per day, and **4**·0.1 machines per day represent a needs-based performance structure.

Now an employment matrix which is not supposed to be optimal yet, but is determined arbitrarily shall be given. As an easy example, every productivity group should be included in equal proportions in every profession in the beginning. And the number of employees in every profession should also be the same. Thus in the beginning the employment matrix ac is given by

$$ac = \begin{bmatrix} 2.5 & 2.5 & 2.5 & 2.5 \\ \vdots & \vdots & \vdots & \vdots \\ 2.5 & 2.5 & 2.5 & 2.5 \end{bmatrix}$$

Hence all primary parameters are given.

The next task would be to determine an optimal employment matrix. But this is only possible after knowing the next paragraphs. First, however, some secondary parameters can be calculated for this demonstration example, which are later applicable for evaluating our employment matrix.

The normalized productivities pn_{j3} are calculated according to equation **(4)** (see page 7) and result in the following values:

$$pn = [\ 2.45\ \ 2.00\ \ 1.65\ \ 2.30]$$

The current mean productivities pm_{j3} are calculated according to equation **(7)** (see page 7) and result in the following values with the currently defined employment matrix:

$$pm = [\ 2.45\ \ 2.00\ \ 1.65\ \ 2.30]$$

pn and pm are currently the same, because initially all productivity groups are represented equally in all professions. As a result, the factors fp_{j3} of the increases of productivity in the individual professions are all equal to one.

For $fp_{j3} = pm_{j3} / pn_{j3}$ applies $fp_1=1$ $fp_2=1$ $fp_3=1$ $fp_4=1$

That means with the given employment matrix there is still no increase in productivity compared to the normalized productivities pn.

The normalized needs-based performance structure is calculated using equation **(9)** (see page 8) and results in the following values:

$$dbn = [\ 0.3722\ \ 0.3722\ \ 0.5583\ \ 0.7444]$$

The components of the vector d of the overall societal work performance are calculated according to equation (**5**) (see page 7) and result in the values:

$$d\ \ = [\ 61.25\ \ \ 50.00\ \ \ \underline{41.25}\ \ \ 57.50\ ]$$

The vector d contains the maximum needs-conform vector db:

$$db\ \ = [\ 27.50\ \ \ 27.50\ \ \ \underline{41.25}\ \ \ 55.00\ ]$$

The comparison of the vector d with the vector db shows that only the performances in profession j3=3 match. That means that currently only the workers in profession j3=4 work for a needs-based performance structure, while in the other three professions there is a relative oversupply of labor. By fluctuation of some workers from the first three professions to the fourth profession, for example, the vector db could be increased and the relative oversupply of workers in the first three professions could be reduced.

The factors fb_{j3} for assessing the needs-conformity of the work performance in the individual professions are calculated according to equation **(11)** (see page 8)

$$fb_1 = \frac{db_1}{d_1} = \frac{27.50}{61.25} = 0.4490 \qquad fb_2 = \frac{27.50}{50.00} = 0.5500 \qquad fb_3 = \frac{41.25}{41.25} = \underline{1.0000} \qquad fb_4 = \frac{55.00}{57.50} = 0.9565$$

They show that society as a whole is still a long way from being able to work according to the needs.

The factor fbg, which is a measure of the needs-conformity of the work performance for society as a whole is calculated by equation **(12)** (see page 8) and results in the value

$$fbg = 0.739$$

The factor fg of the increase in performance for the entire society is calculated by one of the m3 equations **(10)** (see page 8).

$$fg = \frac{db_1}{a \cdot dbn_1} = \frac{db_2}{a \cdot dbn_2} = \frac{db_3}{a \cdot dbn_3} = \frac{db_4}{a \cdot dbn_4} \qquad (13)$$

$$fg = \frac{27.50}{100 \cdot 0.3722} = \frac{27.50}{100 \cdot 0.3722} = \frac{41.25}{100 \cdot 0.5583} = \frac{55.00}{100 \cdot 0.7444} = 0.74$$

This complex factor results from the respective increases in productivity in the various professions and from the needs-conformity of the respective professions, which in turn all depend on the employment matrix ac. It applies fg=fbg, since the mean productivities pm_{j3} in our example are all equal to the normalized productivities pn_{j3} and thus all $fp_{j3}=1$. This example already shows that an inappropriate choice of the employment matrix ac can also lead to a deterioration in performance for society as a whole. In our case, the deterioration in performance was caused solely by work performance in the individual professions that was not needs-conform.

5 The optimal employment structure for society as a whole

Before a corresponding labor market is modeled the optimal employment matrix should be calculated theoretically.

5.1 Criterion of an optimal employment structure

Apart from the employment matrix ac, which can be changed by individual fluctuation or by employment policy measures, all other primary parameters for describing an employment structure are constant over time according to the assumptions formulated above. Thus for now the employment matrix is the only target/objective for optimization.

In a national economy it is in the consumers' interest that the work capacity of all workers is used in a way that a maximum needs-based range of goods and services is generated through an optimal employment matrix ac. The optimality criterion for an employment matrix ac can thus be specified:

An employment matrix ac is optimal if it results in a maximum needs-conform performance vector db.

This is exactly the case when the factor fg, which has already been introduced and represents the overall societal increase in performance, reaches a maximum.

5.2 Solutions for calculating an optimal employment matrix

The optimization of the employment matrix ac leads to a linear optimization problem which can be solved with the simplex algorithm. General statements on the simplex algorithm can be found in Section 4.2.1 of Volume 1 **[4]**.

We are looking for a maximum needs-conform performance structure db depending on the unknown variables $ac_{j2,j3}$, which are the components of the employment matrix.

According to equation **(10)** (see page 8) for each component db_{j3} of the needs-conform performance structure applies

$$db_1 = fg \cdot a \cdot dbn_1 \quad \dots \quad db_{j3} = fg \cdot a \cdot dbn_{j3} \quad \dots \quad db_{m3} = fg \cdot a \cdot dbn_{m3}$$

The number a of workers and the normalized needs-based performance structure dbn are predetermined and cannot be influenced. In order to maximize db the factor fg which is contained in all m3 equations has to be maximized.

Each of the m3 equations can therefore be used as a linear objective function because the only possible maximum for fg results in an overall maximum vector db according to these m3 equations. We take the first equation.

If one assumes that with an optimal employment matrix ac the performance vector d and the included needs-based performance vector db must be identical because no manpower can be wasted, then equation **(5)** (see page 7) can be inserted into the first equation. And so the linear objective function for the main problem is

$$-fg = -\frac{1}{a \cdot dbn_1} \cdot \sum_{j2=1}^{m2} p_{j2,1} \cdot ac_{j2,1} = \text{minimum} \qquad (14)$$

Now, by formulating suitable side conditions it must be ensured that the internal regularities associated with the employment matrix are also taken into account.

First of all, these are the m3-1 equations listed above which are not used as an objective function and therefore have to be entered in a different form as side conditions. They have an internal connection via the constant factor fg·a, which can also be written like this

$$fg \cdot a = \frac{db_1}{dbn_1} = \frac{db_2}{dbn_2} \quad \dots \quad \frac{db_1}{dbn_1} = \frac{db_{j3}}{dbn_{j3}} \quad \dots \quad \frac{db_1}{dbn_1} = \frac{db_{m3}}{dbn_{m3}} \qquad (15)$$

Equation (5) can also be inserted into the m3-1 equations. Accordingly, it can be formulated that for j3 = 2 to m3 applies

$$\sum_{j2=1}^{m2} \frac{p_{j2,1}}{dbn_1} \cdot ac_{j2,1} - \sum_{j2=1}^{m2} \frac{p_{j2,j3}}{dbn_{j3}} \cdot ac_{j2,j3} = 0 \qquad (16)$$

with which m3-1 side conditions are given.

The fact that the sizes ap_{j2} of the productivity groups are given has to be taken into account by further m2 side conditions, so that for full employment applies

$$\sum_{j3=1}^{m3} ac_{j2,j3} = ap_{j2} \tag{17}$$

with which m2 further side conditions are given.

Thereby the optimization problem is already fully described. A simplex table which has m2·m3 unknown variables, m2+m3-1 linear side conditions and one linear objective function can be written for the main problem. Since all side conditions are in non-canonical form, m2+m3-1 auxiliary variables have to be introduced and thus the auxiliary problem has to be solved first.

Then the main problem can be solved if the objective function of the auxiliary problem became zero. If the solution of the auxiliary objective function does not become zero, there is no solution to the main problem.

A TurboPascal program was written for the numerical solution of the optimization problem with the simplex algorithm, so that the numerical solution of the optimal employment structure can also be given for our demonstration example.

5.3 Test calculation with the demonstration example

The already presented demonstration example initially was preset with any non-optimal employment matrix ac. We can now specify the associated optimal matrix and calculate the other secondary parameters to evaluate the result.

In the following values all values belonging to the optimal solution are marked with the index "optimal" in order to distinguish them from the other values of the non-optimal employment matrix already given above, which will be shown again for comparison.

The optimal employment matrix $ac_{optimal}$ has the values

$$ac_{optimal} = \begin{bmatrix} 0 & 0 & 0 & 10 \\ 0 & 0 & 0.37 & 9.63 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 10 & 0 \\ 8.06 & 0 & 1.94 & 0 \\ 0 & 1.48 & 8.52 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 \end{bmatrix} \qquad ac = \begin{bmatrix} 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 & 2.5 \end{bmatrix}$$

As a reminder for the interpretation of the employment matrix: The first line of $ac_{optimal}$ means that all 10 workers in the first performance group are employed in the fourth occupation.

The standardized productivities pn, the mean productivities $pm_{optimal}$ and the productivity increase factors $fp_{optimal}$ have the values

$$pn = [\,2.45 \;\; 2.00 \;\; 1.65 \;\; 2.30\,] \qquad \equiv \qquad pn = [\,2.45 \;\; 2.00 \;\; 1.65 \;\; 2.30\,]$$

$$pm_{optimal} = [\,8.00 \;\; 5.61 \;\; 1.90 \;\; 4.35\,] \qquad pm = [\,2.45 \;\; 2.00 \;\; 1.65 \;\; 2.30\,]$$

$$fp_{optimal} = [\,3.26 \;\; 2.81 \;\; 1.15 \;\; 1.89\,] \qquad fp = [\,1.00 \;\; 1.00 \;\; 1.00 \;\; 1.00\,]$$

It is shown that the optimization in our example has in some cases resulted in considerable increases of productivity in several professions. It is lowest in the professions in which a strong selection is not possible due to the high demand for workers, as can be seen in occupation j3=3.

The overall societal work performance $d_{optimal}$, the included needs-conform proportions $db_{optimal}$ and the factors $fb_{optimal}$ of needs-conformity have the values

$$d_{optimal} = [\,\underline{64.44} \;\; \underline{64.44} \;\; \underline{96.67} \;\; \underline{128.89}\,] \qquad d = [\,61.25 \;\; 50.00 \;\; \underline{41.25} \;\; 57.50\,]$$

$$db_{optimal} = [\,\underline{64.44} \;\; \underline{64.44} \;\; \underline{96.67} \;\; \underline{128.89}\,] \qquad db = [\,27.50 \;\; 27.50 \;\; \underline{41.25} \;\; 55.00\,]$$

$fb_{optimal}$ = [<u>1.00</u> <u>1.00</u> <u>1.00</u> <u>1.00</u>] fb = [0.45 0.55 <u>1.00</u> 0.96]

As expected in the optimal solution applies $d_{optimal} = db_{optimal}$, that means all workers are involved in all professions in the overall societal needs-based work performance. This is a necessary but not a sufficient condition for actually achieving the optimum.

At last, two complex evaluation parameters should be added. These are the factor fbg of the needs-conformity of the overall societal work performance and the factor fg of the overall societal increase in performance

$fbg_{optimal} = 1.00$ fbg = 0.74

$fg_{optimal} = 1.73$ fg = 0.74

As expected, the factor $fbg_{optimal}$ equals one, since all $fb_{optimal}$ already equaled the values one.

The most important evaluation factor $fg_{optimal}$ tells us that by the optimal employment matrix $ac_{optimal}$, which means by an optimal employment policy compared to our arbitrary initial employment matrix ac with fg=0.74, an increase in the overall societal performance of 2.33 is possible.

$db_{optimal} / db = fg_{optimal} / fg = 1.73 / 0.74 = \mathbf{2.33}$

This increase in performance is only based on the optimal use of the workforce, in which everyone does what one can do best relative to others, taking into account the needs of society as a whole. Nobody has to exert themselve more than normal, because normal exertion for all workers in every profession was assumed within the matrix p of individual productivities.

The fact that this solution really represents the optimum is not quite apparent in the results. Only the simplex algorithm guarantees that it is the optimum. In rare cases there may be some other employment matrices that lead to the same increase in performance.

6 Representation of the labor market

So far, the employment structure as a subsystem of an economy has been described mathematically by a system of matrices and vectors. However, this is only an extensive data set that describes a state in general. In addition, a mathematical solution algorithm was used to calculate the data set that describes the optimal state of the employment structure that society should strive for.

But the crucial question is: How has the subsystem of the production conditions to be designed for the system to move towards this optimal employment matrix, preferably by a self-optimizing system?

A well-known subsystem of which this performance is more or less expected is the labor market. It exists with some differences in the capitalist market economy and in the real-socialist planned economy.

Subsequently I want to present a labor market in a mathematically idealized form and examine how it reacts in relation to our requests for self-optimization.

To achieve this it is necessary to phrase rules according to which fluctuations of the workers are possible and thereby enable changes of the current, not yet optimal employment structure (employment matrix). In addition, the driving forces that cause the corresponding changes must be described.

6.1 Mathematical model for simulating the labor market

Wage system:

- A **piece wage Ls_{j3}** is determined for each profession j3.
- If a worker of any performance group j2 in the profession j3 provides a work performance corresponding to its individual productivity $P_{j2,j3}$, then one receives the **individual performance wage $Ld_{j2,j3}$** with

  $$Ld_{j2,j3} = P_{j2,j3} \cdot Lds_{j3} \qquad (18)$$

  In this way, the principles of "equal pay for equal work" and "everyone according to one's performance" are implemented within one profession.
- With the respective normal Md_{j3} for work performance in profession j3, the normal **Mp** for prices (e.g. Mp=1000 DM) and the **normal Mt for time** (e.g. Mt=1 day), the **dimensionless piece wage ls_{j3}** and the **dimensionless individual performance wage $ld_{j2,j3}$** can be introduced

  $$ls_{j3} \;{}_{def}\!= Ls_{j3} \cdot Md_{j3} \cdot Mt / Mp \qquad (19)$$

$ld_{j2,j3\ \ def} = Ld_{j2,j3} \cdot Mt / Mp$ (20)

- If the optimal employment structure has not yet been achieved, there is an apparent oversupply of labor in some professions. The extent of the apparent oversupply is given by the factors fb_{j3} of the needs-conformity of the work performance in the respective professions. If one assumes that there is shortened work in the respective professions without wage compensation, then the **actual individual wage $l_{j2,j3}$** for the worker in productivity group j2 in profession j3 is given by

  $l_{j2,j3} = fb_{j3} \cdot ld_{j2,j3} = fb_{j3} \cdot p_{j2,j3} \cdot ls_{j3}$ (21)

  The m2·m3 individual wages $l_{j2,j3}$ are combined to form the matrix l.

This wage system is the driving force for adapting the employment structure. Every worker can assess in which occupation every person has the **best earnings $lmax_{j2}$** on the basis of the current piece wages ls, the assessment of one's individual abilities p_{j2} and the employment situation in the professions. The worker is motivated to change profession depending on where one earns the most.

Fluctuations:

- Within a certain period of time (e.g. during a reproduction cycle i4), the r1th share of all workers switch to the professions in which they currently would earn the most based on the current piece wages, their individual skills and the employment situation - if they are not already employed in this profession. The factor **r1** is called the **fluctuation rate**. Its size must be determined empirically.

Wage adjustment:

- Between each fluctuation period (reproduction cycle), the piece wages are expressed by the factors fb_{j3} in accordance with the employment situation and corrected using the following formula

  $ls_{j3,i4+1} = ls_{j3,i4} \cdot [1\text{-}r2 \cdot (1\text{-}fb_{j3})]$ (22)

  The factor **r2** is a **parameter of the speed of wage adjustment**. Its size must be identified empirically.

- With every wage correction, the wage level is also brought back to the average value of one.

The fluctuations of workers can now be simulated over any number of reproduction cycles.

Hereby this simple model of a labor market is already described completely. For the computational simulation of the labor market according to this model, a TurboPascal program was written so that our demonstration example can be used to test whether the employment structure is self-optimizing and which optimal wage system occurs.

6.2 Test calculations with the demonstration example

A test calculation has already been executed with the demonstration example mentioned earlier. In addition to the predefined time-invariant parameters “productivity matrix p”, “productivity group sizes ap” and “needs-based performance structure db0 or dbn” the non-optimal employment matrix ac was assumed as the initial parameter. A vector ls of piece wages, which serve as initial values has to be added to this data record. This vector ls of piece wages does not have to be optimal since self-optimization is expected within the simulation calculation, too. An arbitrary starting vector was determined by

$ls_{i4=0} = [\,1 \quad 1 \quad 1 \quad 1\,]$

Furthermore, the empirical factors of the fluctuation rate r1 and the speed of price adjustment r2 must be given. It was assumed

r1 = 0.3 r2 = 0.3

Over n4=300 reproduction cycles have been calculated. In our example the model of the labor market actually converges to the optimal employment matrix $ac_{optimal}$, as we have already calculated in another way. After approximately 100 reproduction cycles the maximum increase in performance and the needs-based work performance are achieved. The maximum increase in performance is represented using the factor fg (and separately using the factors fp_{j3}). The needs-based work performance is represented using the factor fbg (and separately using the factors fb_{j3}).

$fg_{n4=0}$ = **0.74** $fp_{n4=0}$ =[1.00 1.00 1.00 1.00] $fbg_{n4=0}$ = 0.74 $fb_{n4=0}$ =[0.45 0.55 1.00 0.96]

$fg_{n4=300}$= **1.73** $fp_{n4=300}$=[3.26 2.81 1.15 1.89] $fbg_{n4=300}$= 1.00 $fb_{n4=300}$=[1.00 1.00 1.00 1.00]

Figure 1: Development of the factor fg for increase of overall societal performance and the factor fbg for overall societal needs-conformity

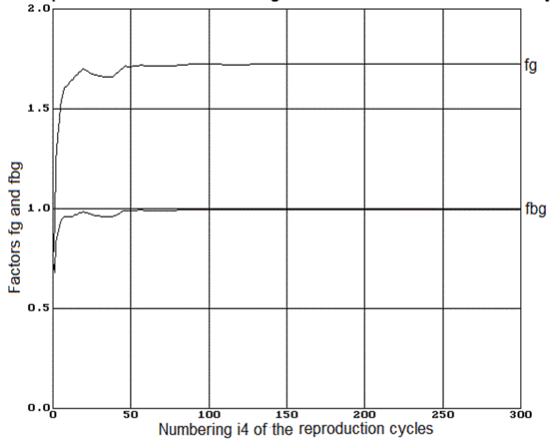


Figure 2: Development of the factors fp_{j3} of productivity increases of work in the professions j3

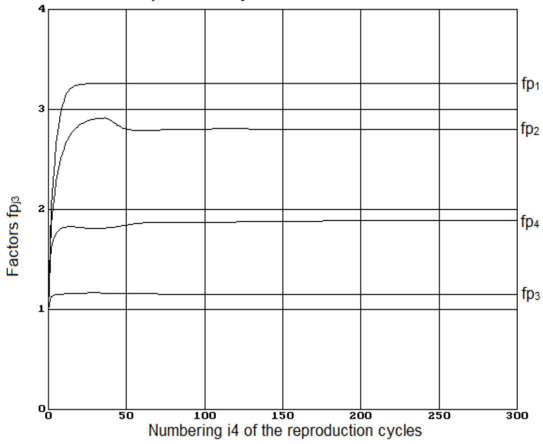


Figure 3: Development of the factors fb_{j3} of the needs-based shares of work in the professions j3

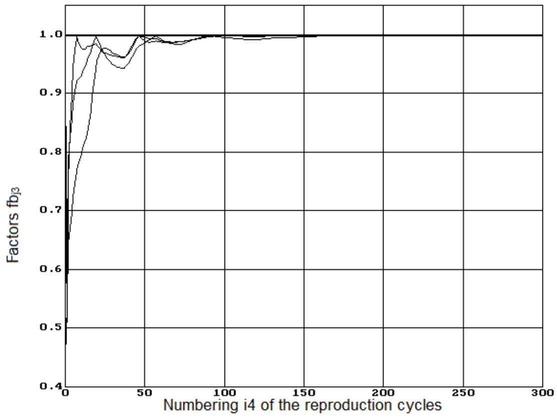
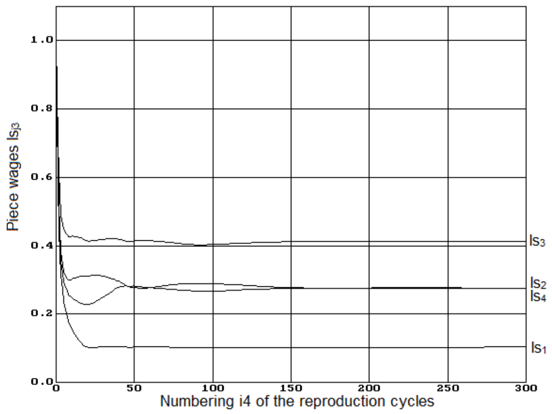


Figure 4: Development of the piece wages ls_{j3} in the professions j3



In **Figures 1** to **3** diagrams demonstrate these factors in their time course from n4= 1 to 300 and thus demonstrate the convergence behaviour of the model by this example.

The piece wages ls_{j3} also converge to optimal values. Here, however, the optimal values are only reached after approx. 150 reproduction cycles, as can be seen in **Figure 4**. After 300 reproduction cycles the piece wages $ls_{i4 = 300}$ had reached the following almost optimal values

$$ls_{opt} \approx ls_{i4=300} = [\ 0.103 \ \ 0.276 \ \ 0.414 \ \ 0.276]$$

The simulation program also provides the matrix of the individual wages $l_{i4 = 300}$ after 300 reproduction cycles. Together with the employment matrix $ac_{i4 = 300} \approx ac_{optimal}$, which corresponds to the optimal employment matrix, it shows which wages the workers actually receive in their current professions and also which wages they are (would be) receiving in other professions at the same time.

The realized wages are highlighted in bold in the matrix $l_{i4 = 300}$ given below. In addition the associated employment matrix is also given here.

$$l_{optimal} \approx l_{i4=300} = \begin{bmatrix} 0.10 & 0.28 & 0.83 & \mathbf{1.10} \\ 0 & 0 & \mathbf{0.83} & \mathbf{0.83} \\ 0.31 & 0.55 & \mathbf{0.83} & 0.41 \\ 0.21 & 1.10 & \mathbf{1.24} & 0.69 \\ 0.10 & 0.28 & \mathbf{0.41} & 0 \\ \mathbf{0.83} & 0.55 & \mathbf{0.83} & 0.55 \\ 0.26 & \mathbf{0.83} & \mathbf{0.83} & 0.28 \\ 0.62 & \mathbf{1.66} & 0.41 & 0.28 \\ 0 & 0 & 0 & \mathbf{1.66} \\ 0.10 & 0.28 & \mathbf{0.62} & 0.55 \end{bmatrix} \qquad ac_{optimal} \approx ac_{i4=300} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{0.38} & \mathbf{9.62} \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ \mathbf{8.06} & 0 & \mathbf{1.94} & 0 \\ 0 & \mathbf{1.49} & \mathbf{8.51} & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{10} & 0 \end{bmatrix}$$

The remarkable result shows that there is actually an optimal system of piece wages, which ensures that all workers in the profession in which they are working in the interests of an overall societal maximum needs-based work actually receive the highest individual wages without exception. Accordingly, they should endeavor to work precisely in the respective profession, given they are sufficiently informed. What is also interesting about the result is that the system has led to optimal unit wages and thus to optimal individual wages in a self-optimizing way simply by a very simple principle of wage adjustment according to supply and demand.

7 Some selected analyses to demonstrate the capabilities of the model

7.1 Change in the performance evaluation between the individual professions by changing the performance needs structure

A variation of the demonstration example will show how the individual performance evaluation changes due to a change in the structure of the overall societal needs for work performance. The only modified parameter in the needs-based performance structure db0 is parameter $db0_1$, which is altered from 2 to 4

$$db0_{new} = [\ \mathbf{4} \quad 2 \quad 3 \quad 4\ ] \qquad db0_{old} = [\ 2 \quad 2 \quad 3 \quad 4\ ]$$

The simulation calculation over n4=300 cycles converges similarly to the original example. It of course results in another optimal employment matrix ac

$$ac_{optimal,new} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{5.40} & \mathbf{4.60} \\ \mathbf{4.35} & 0 & \mathbf{5.65} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ \mathbf{10} & 0 & 0 & 0 \\ 0 & \mathbf{5.88} & \mathbf{4.12} & 0 \\ \mathbf{3.46} & \mathbf{6.54} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{10} & 0 \end{bmatrix} \qquad ac_{optimal,old} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{0.38} & \mathbf{9.62} \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ \mathbf{8.06} & 0 & \mathbf{1.94} & 0 \\ 0 & \mathbf{1.49} & \mathbf{8.51} & 0 \\ 0 & \mathbf{10} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{10} & 0 \end{bmatrix}$$

This also results in different optimal piece wages $ls_{optimal}$ and different individual wages $l_{optimal}$.

$ls_{optimal,new}$ = [0.242 0.242 0.364 0.242] $ls_{optimal,old}$ = [0.103 0.276 0.414 0.276]

$$l_{optimal,new} = \begin{bmatrix} 0.24 & 0.24 & 0.73 & \mathbf{0.97} \\ 0 & 0 & \mathbf{0.73} & \mathbf{0.73} \\ \mathbf{0.73} & 0.48 & \mathbf{0.73} & 0.36 \\ 0.48 & 0.97 & \mathbf{1.09} & 0.61 \\ 0.24 & 0.24 & \mathbf{0.36} & 0 \\ \mathbf{1.94} & 0.48 & 0.73 & 0.48 \\ 0.61 & \mathbf{0.73} & \mathbf{0.73} & 0.24 \\ \mathbf{1,45} & \mathbf{1.45} & 0.36 & 0.24 \\ 0 & 0 & 0 & \mathbf{1.45} \\ 0.24 & 0.24 & \mathbf{0.55} & 0.48 \end{bmatrix} \quad l_{optimal,old} = \begin{bmatrix} 0.10 & 0.28 & 0.83 & \mathbf{1,10} \\ 0 & 0 & \mathbf{0.83} & \mathbf{0.83} \\ 0.31 & 0.55 & \mathbf{0.83} & 0.41 \\ 0.21 & 1.10 & \mathbf{1.24} & 0.69 \\ 0.10 & 0.28 & \mathbf{0.41} & 0 \\ \mathbf{0.83} & 0.55 & \mathbf{0.83} & 0.55 \\ 0.26 & \mathbf{0.83} & \mathbf{0.83} & 0.28 \\ 0.62 & \mathbf{1.66} & 0.41 & 0.28 \\ 0 & 0 & 0 & \mathbf{1.66} \\ 0.10 & 0.28 & \mathbf{0.62} & 0.55 \end{bmatrix}$$

A comparison between the realized individual wages (printed in bold) and the potential wages shows that, as expected, all potential wages are lower than the realized ones, so that no worker in this optimal state of the employment matrix aspires to change profession and thus to worsen the optimal employment matrix.

What is more interesting, however, is the comparison of wages between the old and the new example. The same work within one profession will only be valued differently due to different needs of society. This shows an essential difference between the concept of performance in physics and the concept of performance in the field of societal work. In physics, the conversion factors between various performances or forms of energy are always the same. Due to the need to optimize the overall societal employment structure there is a comparability of different work performances via the quantities of individual wages within a labor market in societal work. However these comparisons are only relative since they always depend on the current level of the productive forces' development in society as a whole.

This example is evidence that the Darwinian ideology that the stronger (= better = more efficient) wins and the weaker does not deserve to survive is not applicable to a human society with joint production. This argument is often used by those who have just found a more favourable position in the distribution of social goods. With this ideology, it is tacitly assumed that human performance, similar to physical performance, is a fixed quantity in itself and whoever cannot achieve this performance or can only achieve this to a small extent would be once and for all inferior or useless to society. That is obviously wrong. In a society with collective work the spectrum of different productivity profiles brings the possibility of increasing productivity for society as a whole.

7.2 A new profession is arising

As the productive forces develop due to the development of new products and production technologies new professions arise as well. Another example will show how a new profession can affect the wage system. The original demonstration example is used again. The matrix of individual productivities is expanded by one profession j3=5. The number m2 of performance groups remains constant.

$$\begin{array}{ccc} \text{j3:} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & \mathbf{5} \end{array} & \text{j2} \\ p_{new} = & \begin{bmatrix} 1 & 1 & 2 & 4 & \mathbf{0.4} \\ 0 & 0 & 2 & 3 & \mathbf{0.6} \\ 3 & 2 & 2 & 1.5 & \mathbf{1} \\ 2 & 4 & 3 & 2.5 & \mathbf{0.3} \\ 1 & 1 & 1 & 0 & \mathbf{0.7} \\ 8 & 2 & 2 & 2 & \mathbf{0.9} \\ 2.5 & 3 & 2 & 1 & \mathbf{0.2} \\ 6 & 6 & 1 & 1 & \mathbf{0.5} \\ 0 & 0 & 0 & 6 & \mathbf{0.6} \\ 1 & 1 & 1.5 & 2 & \mathbf{0.8} \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \end{array}$$

In addition, the needs vector db0 is expanded by a 5th component.

$db0_{new} = [\,2 \quad 3 \quad 3 \quad 4 \quad \mathbf{1}\,]$

All the other primary parameters remain the same.

The simulation calculation over n4=300 cycles converges similarly to the original example. It results in a different optimal employment matrix ac, of course

$$ac_{optimal,new} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{9.25} & \mathbf{0.75} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{10} \\ \mathbf{5.28} & 0 & \mathbf{4.09} & 0 & \mathbf{0.63} \\ 0 & 0 & \mathbf{10} & 0 & 0 \\ \mathbf{1.48} & \mathbf{8.52} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{10} \end{bmatrix} \qquad ac_{optimal,old} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{0.38} & \mathbf{9.62} \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ 0 & 0 & \mathbf{10} & 0 \\ \mathbf{8.06} & 0 & \mathbf{1.94} & 0 \\ 0 & \mathbf{1.49} & \mathbf{8.51} & 0 \\ 0 & \mathbf{10} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{10} \\ 0 & 0 & \mathbf{10} & 0 \end{bmatrix}$$

This also results in different optimal piece wages $ls_{optimal}$ and different individual wages $l_{optimal}$.

$ls_{optimal,new} = [0.110\ 0.110\ 0.440\ 0.293\ 0.978] \qquad ls_{optimal,old} = [0.103\ 0.276\ 0.414\ 0.276]$

$$l_{optimal,new} = \begin{bmatrix} 0.11 & 0.11 & 0.88 & \mathbf{1.17} & 0.39 \\ 0 & 0 & \mathbf{0.88} & \mathbf{0.88} & 0.59 \\ 0.33 & 0.22 & 0.88 & 0.44 & \mathbf{0.98} \\ 0.22 & 0.44 & \mathbf{1.32} & 0.73 & 0.29 \\ 0.11 & 0.11 & 0.44 & 0 & \mathbf{0.68} \\ \mathbf{0.88} & 0.22 & \mathbf{0.88} & 0.59 & \mathbf{0.88} \\ 0.28 & 0.33 & \mathbf{0.88} & 0.29 & 0.20 \\ \mathbf{0{,}66} & \mathbf{0.66} & 0.44 & 0.29 & 0.49 \\ 0 & 0 & 0 & \mathbf{1.76} & 0.59 \\ 0.11 & 0.11 & 0.66 & 0.59 & \mathbf{0.78} \end{bmatrix} \qquad l_{optimal,old} = \begin{bmatrix} 0.10 & 0.28 & 0.83 & \mathbf{1.10} \\ 0 & 0 & \mathbf{0.83} & \mathbf{0.83} \\ 0.31 & 0.55 & \mathbf{0.83} & 0.41 \\ 0.21 & 1.10 & \mathbf{1.24} & 0.69 \\ 0.10 & 0.28 & \mathbf{0.41} & 0 \\ \mathbf{0.83} & 0.55 & \mathbf{0.83} & 0.55 \\ 0.26 & \mathbf{0.83} & \mathbf{0.83} & 0.28 \\ 0.62 & \mathbf{1.66} & 0.41 & 0.28 \\ 0 & 0 & 0 & \mathbf{1.66} \\ 0.10 & 0.28 & \mathbf{0.62} & 0.55 \end{bmatrix}$$

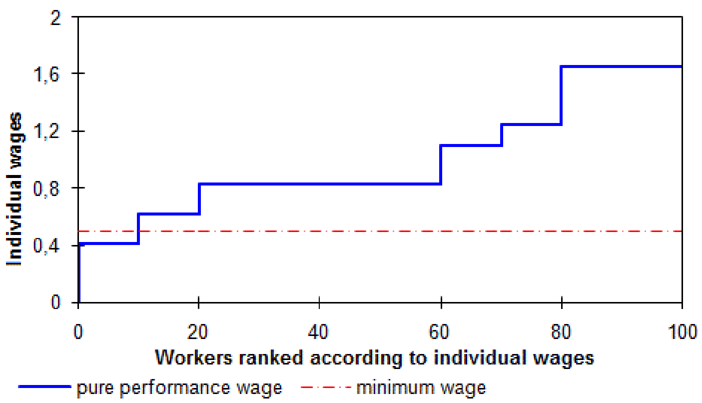
By introducing the new profession the 8th performance group becomes the big loser. Their realized individual wages are reduced to less than half, while all other performance groups improve slightly. It must be emphasized that this change is only due to a change in the performance evaluation and not due to a lower actual work performance. The workers in performance group 8 in profession 2 previously received a wage of 1.66. After the introduction of the new profession they only receive a wage of 0.66 for the same work, while the societal wage level remained constant at 1. This revaluation of performance may seem unfair to the individual. However, it is not an arbitrary or subjective assessment, but rather objectively determined by the labor market, whose task it is to create an optimal employment structure.

The parameters of this example were chosen arbitrarily in order to demonstrate the possible dynamics of a performance evaluation in principle. The extent to which such developments will affect the real economy cannot yet be said. It requires further investigation with realistic parameters and a considerably larger calculation effort. This example must also not serve to justify considerable wage differences and fluctuations in the real capitalist market economy, since there is a lot of arbitrariness due to a lack of theoretical foundations. The aim of these investigations is to reduce this arbitrariness. I see my investigation only as one step on the way to a satisfactory solution to the problem of the objectively justified and thus fair individual performance evaluation of human work.

7.3 Introduction of a social component into the remuneration system

So far, considerations have been only made as to how a labor market can be designed with a performance-oriented wage system so that every worker can be motivated to achieve a maximum performance within a profession and at the same time strive for a profession which leads to a needs-based performance structure with a maximum productivity for society as a whole. However, we have not yet given any thought to whether the result is also socially acceptable.

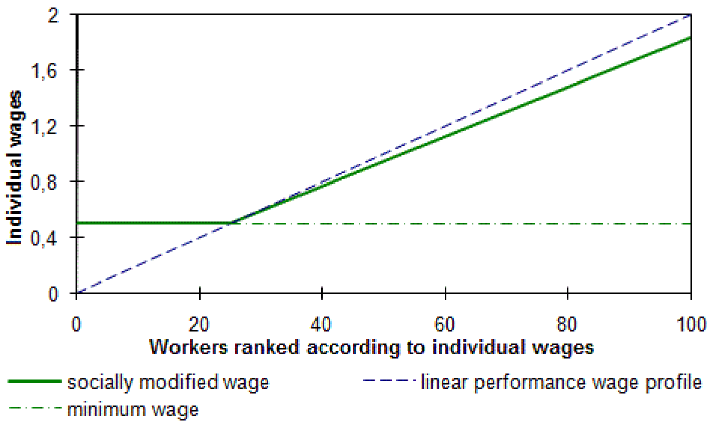
Figure 5: Wage structure of the pure performance wage for the demonstration example



Using our original demonstration example we can represent the wage or income profile for a society consisting of 100 workers by drawing a diagram in which for each worker this wage is plotted against the number of workers – ordered by the size of its individual wage. This creates the Figure of the wage structure in society. This is shown in **Figure 5**. The area under this curve (solid line) represents the total societal wage volume. Let us assume that the necessary wage to buy the essential goods for survival is 0.5 (dash-dot line). In our demonstration example there is a performance group consisting of 10 workers who are not able to cover their essential needs for consumer goods with their performance wages.

In a social economic system, every member of society must be in a position to afford at least essential consumer goods. In a market economy wages are the main and usually the only source of income for most members of society. Accordingly, in a social market economy it must be ensured that every member of society who is capable of working can earn its necessary maintenance through individually realizable wages. This makes it necessary that for every work in full-time a corresponding minimum wage has to be paid.

Figure 6: First modification of a linear performance wage profile



Accordingly, the wage system modelled so far is not yet socially acceptable. So it has to be considered how this wage system can be modified without losing the performance orientation.

To simplify the illustration let's assume that the income profile is a straight line starting with the minimum wage equal to zero and ending with the maximum wage equal to two, and the necessary minimum wage is 0.5 (dash-dot line), according to **Figure 6** (dashed line). This means that 25% of the workers cannot completely cover their vital needs.

The federal German social system works in the way that the members of society with income above the social welfare level raise the deficit through taxes. This money is redistributed to the people in need via the social welfare office. If one assumes, for the sake of simplicity, that the solidarity tax is proportional to the wage share that is above the social welfare level, the result is an income profile as shown in **Figure 6** (solid line).This results in a corrected maximum income of 1.83.

The disadvantage of this solution is that 25% of the workers do not have any positive incentive to perform. This means that for all these workers it is ensured through social control and regulation (negative performance incentive = compulsion) that they bring their performance potential, albeit low, into the overall social work performance. (Provided that the society offers them a job, which is not to be discussed here.)

Figure 7: Second modification of a linear performance wage profile

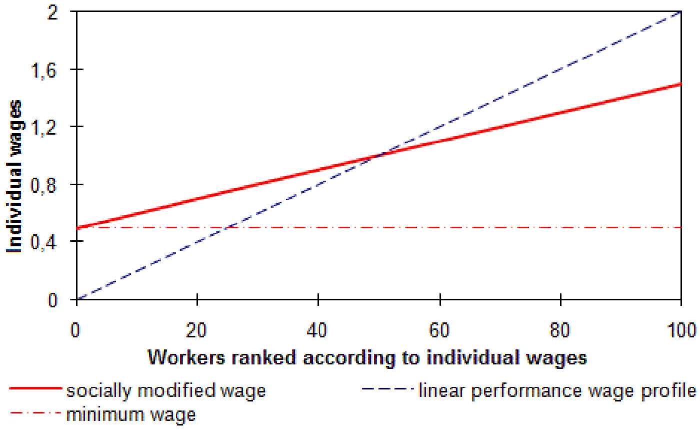
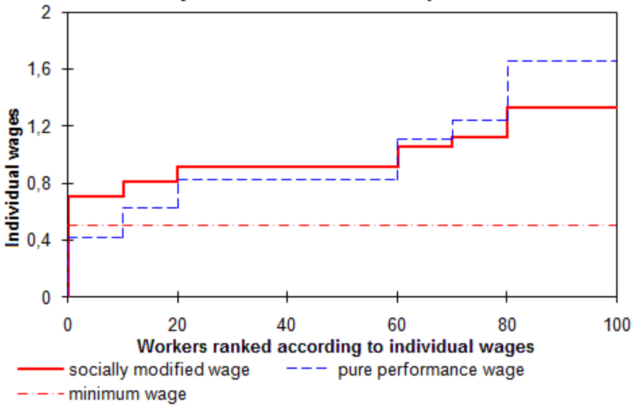


Figure 8: Socially modified wage structure of society for the demonstration profile



Instead, I would suggest modifying the incentive pay system as shown in **Figure 7**. There is positive motivation to perform right from the start. This system would be easier to implement from an administrative point of view. In our example, the peak income continues to decrease from 1.83 to 1.5. But I don't see a problem in this since for top earners the largest part of the income keeps being performance-related (in our example it is still 2/3). Problems could only arise in the lower income groups.

The application of this suggestion for a social modification of the performance wage system to our demonstration example is shown in **Figure 8** (solid line). In addition, the income profile of the plain performance wage was entered as a dashed line and the necessary minimum wage as a dash-dotted line.

These considerations show that a performance wage system in a social market economy, and thus the market economy as a whole, can only function if a surplus product for society as a whole is generated which allows additional consumption to what is existentially necessary. The more favourable the relationship between additional and necessary consumption, the more effective the performance principle is likely to be. A prerequisite is, however, that the social surplus product is distributed via a relating performance wage system and is not withdrawn from the cycle of consumption through private capitalist appropriation.

8 Integration of the labor market model into a model of an entire national economy

The model of the labor market naturally describes only a part of an economic system – the one which was idealized in the models in Volume 1 [4] via the aggregation of all working forces. As the simulation of fluctuating working forces has worked so well with my model it is a tempting task to integrate the labor market model with the models of national economies. Since this task, including the sub-problems that are still to be expected, means a greater effort, I have to postpone it. But I can't help but at least take the first step and then briefly discuss some expected problems.

8.1 Representation of the productive forces of an economy independently of the conditions of production [1]

Analogous to Volume 1 [4], the societal productive forces are now to be described independently of the production conditions. The description has been tightened up. The detailed explanations can be found in the corresponding sections of Volume 1 and previous sections of this Volume and apply accordingly here. Essentially the same names and symbols are used.

Reproductive cycles:

- The reproductive process of society takes place in discrete periods of time **Tz** of equal length, which are called reproductive cycles.

Society's resources:

- The society consists of the number of **a** employable members.
- There are **n3** different types of goods **i3**.
- The society currently has the total amount of economic goods **wg** at its disposal.

Skills of the workers (professions):

- There are **m3** different professions **j3** known in which the workers can be employed.
- All a workers can be assigned to **m2** productivity groups **j2**, each with the same productivity profile.
- The totality of all productivity profiles is described by the productivity matrix **p**.
- The numbers of workers in the respective productivity groups are given by the vector **ap**.

Production processes and producing economic entities:

- There are **n2** production processes or companies **i2**.
- The production process i2 is represented as a black box in the following way: If a nwwds-conform range of goods $\mathbf{wi}_{i2}$ is provided at the beginning of a reproduction cycle and a demand-conform range of work performance $\mathbf{ai}_{i2}$ is applied to the range of goods over the period of this reproduction cycle within the production process i2, the range of goods $\mathbf{wi}_{i2}$ at the end of the reproduction cycle is transitioned to the range of goods $\mathbf{wo}_{i2}$.
- The needs-conform range ai_{i2} of work performance is a vector consisting of m3 components $ai_{i2,j3}$.The component $\mathbf{ai}_{i2,j3}$ indicates how much work is required in the respective profession j3 for the production process i2. The vectors ai_{i2} are combined to form the normal work input matrix **ai**.
- Each multiple x_{i2} of the matrices ai_{i2}, wi_{i2} and wo_{i2} represents the same production process with a different production volume $\mathbf{x}_{i2}$.

This means that all n2 production processes i2 are fully described by the two input matrices ai and wi and the output matrix wo. In contrast to the models in Volume 1 [4], here the labor input vector ai has been expanded to form a labor input matrix ai. While the component ai_{i2} of the vector ai previously indicated a number of fully employed workers, the component $ai_{i2,j3}$ of the matrix ai now contains a quantity of profession-specific work performance, measured in the performance unit j3 of the respective professions.

Employment situation:

- In order to fully describe the employment situation one now needs a three-dimensional employment matrix ac.The component $ac_{i2,\ j2,\ j3}$ indicates how many workers with the productivity profile j2 are employed at company i2 with the profession j3.
- **Alternative:** In order to reduce the flood of data the three-dimensional employment matrix ac could also be replaced by two two-dimensional matrices ac’ and ac’’ with an acceptable loss of information. The matrix ac’ then contains the previously known information, in which the component $\mathbf{ac'}_{j2,\ j3}$ indicates how many workers in productivity group j2 are employed in profession j3, regardless of which company. The matrix **ac’’** then indicates which working capacity the companies have under contract by the component $\mathbf{ac''}_{i2,\ j3}$ indicating how much profession-specific performance potential company i2 has under contract in profession j3.

Which of the variants is ultimately more advantageous has to be shown in practice.

Needs and multiplication of members of society (workers):

- Similar average needs of all workers are assumed and described by the vectors **kni**, **kno**, **kli** and **klo**.
- The multiplication factor of the supplied workers per reproduction cycle is given as **fa**.

Consumption of members of society:

- $\mathbf{ak}_{j2}$ is the number of workers in productivity group j2 who were supplied with the essential consumer goods. The components are combined to form the vector **ak**.
- $\mathbf{fl}_{j2}$ is the current factor of the additional consumption of productivity group j2. The components are combined to form vector **fl**.

This completely describes the system of the productive forces of a national economy, taking into account a differentiated employment structure. The dimensionless parameters were tacitly given without going into detail. In summary, the complete parameter set can now be specified:

i2	Index or number of the production process or company
i3	Index or number of the respective good
n2	Number of the production processes or companies
n3	Number of the different goods
ap	productivity group sizes
ak	numbers of supplied workers in the productivity groups

fa multiplication factor of workers

kni input of the essential consumption
kno output of the essential consumption
kli normalized input of additional consumption
klo normalized output of additional consumption
fl factors of additional consumption in the productivity groups

ai normalized inputs of workforces
wi normalized inputs of means of production
wo normalized outputs of production
x production volumes
ac or ac’, ac’’ employment matrix

wg total quantities of economic goods

This is the minimum set of parameters that is necessary to fully describe the linear discrete model of the productive forces of a national economy with a differentiated employment structure. All other parameters can be derived from it.

8.2 Short discussion for continuing the problem

The formulation of the productive forces has already shown that the amount of data to be obtained and processed has increased considerably. Scalar parameters became vectors, a vector became a matrix, and a two-dimensional matrix became one three-dimensional or two two-dimensional matrices. The further differentiation of the data structure naturally also results in a further multiplicity of variants of the possible relationships between these data. The selection of it can be debated because the reality usually lies somewhere in between. This would become evident when we formulate the conditions of production, which would come next. But I don't want to go down this path any further for now, because it's a huge task.

Perhaps a hint of a little problem as a sample: The multiplication factor fa was given for all members of society. However, it is possible, e.g. through differentiated remuneration of the productivity groups, that different supply rates occur and thus different actual multiplication factors of the productivity groups occur. Does it make sense to let the productivity groups grow differently accordingly, or does the structure of the skills of society (represented by the relationships in the vector ap of the productivity group sizes) remain constant? I would go for the latter. But one can also see it differently.

This is the opportunity to point out a general problem: Generally, everyone tries to develop models that are as accurate as possible in the interest of the accuracy of the representation of reality and to go further and further. In doing so, however, one also runs the risk of losing track of the actual goal, namely applicable insights, when trying to present details correctly. So you have to think carefully about whether or when to go one step further.

Apart from the fact that in the near future I will not have the time to deal with the further processing of the extended model formulated here, it is also not yet time. The informative value and the possible variations of the simpler models have not yet been exhausted. In addition, it is time to draw at least preliminary conclusions for a societal alternative from what we have learned so far.

From the extended model, however, I expect interesting insights into the more complex intrinsic movement of a market economy in the future.

9 Some critical remarks and conclusions on the developed model and outlook for further necessary investigations

The following remarks refer back to the original separate model of the labor market as discussed in detail in Sections 2 to 7.

9.1 The labor market model is a study model

The model is a study model in the sense that no attempt was made to illustrate the labor market found in the real capitalist market economy as precisely as possible. Instead, an attempt was made (of course inspired by the real market) to formulate an idealized model and to investigate by means of simulation calculations whether the principle is suitable for causing self-optimization. This model is intended to provide a profound basis for discussion in the area of the different evaluations of the problems of the existing market and the development of ideas as to what a labor market could look like in a social or socialist market economy.

Accordingly, this model cannot show how a wage level develops in the conflict between the owners of the means of production and the wage earners represented by the employers' association and the trade unions. In my model the nominal wage level is predetermined based on assumptions and is constant over time. Instead of collective bargaining the wage relations between the various professions are altered by a principle of the supply of work in different professions by the workers and the request for a needs-based structure of work by the employers. The wages of the different professions can rise and fall, while the wage level remains the same.[2]

The positive result of this model is the statement that a system of wage relations actually exists for a free labor market, which ensures that every worker has the best income opportunities in exactly the profession where one should also work in the interests of maximum productivity for society as a whole. This means that with this optimal wage system the worker will strive for the right job on its own and not have to be forced to do so by others, e.g. by sacking or by duty.

In addition, another positive result is that this optimal wage system does not have to be calculated by experts using theoretical models, but that it occurs when one slowly shifts the ratio of wages depending on supply and demand, e.g. on the basis of statistical evaluations, by lowering the wages in professions with a relative oversupply and raising the wages in professions with increased demand.

But this can only work if these wage changes cannot be abused by the employers to gradually lower the wage level further and further so that the wage level becomes the battleground of the classes. A free labor market therefore only has a chance in an exploitation-free market economy.

I hope that in a really social market economy with this roughly drafted labor market an instrument can be found with which on the one hand there is sufficient differentiation of wages in order to create a meaningful performance principle, and that, on the other hand, the extreme differentiation of the capitalist market economy can be avoided. However, further investigations are necessary for this.

9.2 Performance evaluation within one profession

My model of the labor market dealt specifically with how the evaluation of qualitatively different jobs in different professions is possible according to objective criteria. Hereby it was assumed that performance within an occupation can be measured. This is the case in some professions. In many other professions, however, this is difficult. These are especially the professions where creative intellectual work is done. Unfortunately, at the moment I can't imagine anything more than that on average correct evaluations are carried out through many subjective individual evaluations in the context of bilateral negotiations between management and employees. But these subjective individual evaluations hold the risk that if there is an asymmetry in economic power, which usually is in favour of the management of a company facing an individual worker, the performance evaluation drifts downwards, and with it the wages as well. So reliable social mechanisms must be installed that maintain the average wage level.

9.3 Performance evaluation of the members of the management of a company

In a capitalist private company all employed workers (including employees and executives) have a supervisor with whom they can negotiate their performance evaluation and thus their wages (at least their classification). Only the top manager has no supervisor with whom to negotiate its performance evaluation and its wages. In capitalism the problem is simply solved. The top manager owns the business and the profit is this person's income. That sounds plausible and practical. But therefore we all have to deal with the problems of the capitalist market economy.

As a socialist, I believe that personal income must result essentially from personal work and not from property. So I also have to think about how the work of the management is judged fairly.

Managing director can be seen as a profession like any other. In terms of the model of self-optimization of the wage relations between the professions one could assume that the principal amount of the manager’s wages is given depending on the size of the company. In order to continue to motivate the manager in its daily work part of its wages must be performance-related. Its performance can only be measured by the company's success. No one is allowed to subjectively assess success here, but rather it has to be done using previously selected success indicators. Such indicators can only be determined much later after the actions that caused them. The actions of the managing directors are not only important for current success, but also have an effect far into the future in the form of strategic decisions. In order to increase the manager's interest in the future of the company a pay mode should be found that ties a considerable portion of the performance-related wage component to future success which of course means that this wage is paid much later. This principle should be socially acceptable for managing directors because it can be assumed that managing directors belong to the upper salary groups even in a social market economy and do not immediately depend on full wages. This can then mean that they still receive income from their work after they have finished their work. But this makes sense if the old managing director is significantly involved in the selection and training of the new managing director and thus creates the conditions for the future success of the company even when the person leaves. In this way, a reliable interest in the success of the company could be generated even without being the owner. This principle could also be applied in stages to other managerial staff or to employees who work in development. Such forms of wages and corresponding experiences with them already exist. This is the case, for example, for the remuneration of inventions made in capitalist companies according to the employee remuneration scheme. However, they only play a very subordinate role in the overall wage volume.

There is still some creative thinking to be done in this area. My model of the labor market is too simplified to contribute to an increase in knowledge in this area. At the moment I have no idea how one could incorporate such important considerations into the modelling.

9.4 Interpretations of the performance matrix

In my demonstration model a performance matrix was given seemingly effortless. It should not only contain information about the performance of all workers in society in their current profession but also about their potential performance in all other professions. It thus represents a huge amount of information. The question now arises whether this matrix or its informations are present in society which means whether it can be gathered in any way. As interesting as this information would be for a model calculation of a real economic system, I am assuming that for the foreseeable future no usable values will be compiled in any information source so that it can be used for scientific or economic-political purposes.

Nevertheless, this information exists in the minds of the members of society and accordingly has an effect on a real labor market. So every young professional thinks about what its special skills are when choosing their profession, if one has a choice and also has more or less reliable information about how much can be earned in the different professions. Even if you are thinking of changing your career, you have to think about it: In which areas do I have which skills - immediately available, after a certain familiarization with the new job or after a possible retraining? The knowledge of one's own abilities thus represents one line in the productivity matrix p. This means that the information in the performance matrix is present in society at the points where it is needed for action, namely for individual fluctuation decisions, so that this information also has an effect on the labor market without anyone knowing the entirety of this information.

However, it must be taken into account that in the case of fluctuation decisions it is not the actual skills that initially have an effect, but the skills reflected in the self-perception which can differ from the actual ones. Only at a much later point in time after starting a new job there is feedback as to whether the expectations have actually been met and whether, in the event of an error, a new fluctuation decision may be necessary. So for the process of optimizing the labor market to work everyone should assess their personal skills as correctly as possible - this is a general interest of society. It also results in an objective interest of society in really good career counseling. This should not be limited to persuading jobseekers to work wherever possible where there is a current labor shortage, but rather to provide help to perceive one's own abilities and inability as far as possible free of economic-political calculations. In addition, this counseling should provide well-founded information about possible income in the various professions.

9.5 Stochastic phenomena on the labor market

In comparison with reality the model used so far has the simplification that once the parameters have been defined they have a very determined effect and lead to definite and exact results. For now there is no room for chance.

However, the real processes underlie random scatters. As shown in the previous section, for example, there are discrepancies between actual abilities and your own assessment of these abilities, for a wide variety of reasons. There is only a certain probability that this will lead to correct fluctuation decisions.

In many professions the evaluation of the actual work performance is still influenced by considerable subjective influences. Thus the correlation between performance and received wages continues to contain a random component.

It is possible to introduce stochastic elements into the model and into the corresponding simulation calculation. At first, however, it seemed to me more reasonable to carry out my investigations with the most exact behaviour of the model in order not to obscure the basic behaviour of the model and the objects to be depicted. It must be left to later investigations to take into account stochastic phenomena, which will certainly lead to some interesting findings.

For example, it could be an interesting question to what extent errors in self-assessment lead away from the societal optimum.

Another interesting question is how big wage differences must be in order for voluntary optimizing fluctuations to take place against the effort and risk of changing jobs. A significant role plays the scattering of the objective or perceived correlation between performance and wages, which can be taken into account by introducing stochastic elements. At this point, however, the collaboration of sociologists is also required.

Conclusion: Both ways are to be followed in the future, both with and without consideration of stochastic model elements. At the beginning, deterministic models are preferable because of the better overview.

9.6 Barriers to fluctuation

Every change of job means additional effort, both for the worker and for the company. This is especially true if you also change your profession. Expenses are the search for a new job, familiarization with the new job or retraining in a new job, work-related change of residence, double housekeeping with at least temporarily different work places of the family members.

These expenses naturally represent barriers to fluctuation that must be overcome by those affected. Therefore, there must also be a sufficient incentive in the form of a noticeably higher income due to fluctuation.

These barriers to fluctuation have not yet been fully taken into account in my model, and further qualifications of the model are still possible. In some cases, however, they have already been taken into account by assuming that only a proportion of workers who could improve their jobs actually change their jobs. This is taken into account by the empirically introduced factor r1, the fluctuation rate.

Variations in the fluctuation rate r1 have also shown that the fluctuation barriers not only have a negative effect in that they inhibit the self-optimization of the labor market. They are also a necessary namely a system-stabilizing element of the labor market. If, for example, the fluctuation rate had the value 1, i.e. every worker who can improve its income would switch immediately, massive fluctuations would set in immediately with the slightest change in the labor market. That would lead to an instable system.

If the fluctuation barriers are better taken into account in the model in the future, a distinction must be made between two fundamentally different barriers. First of all, there are those who represent an inhibition threshold. Nothing happens here at first if there is a small reason for fluctuation. Only when the benefit of a fluctuation has exceeded a threshold value does the person concerned decide to switch. Second, there are those barriers where fluctuations still take place even with the smallest reasons for fluctuation, but the frequency of fluctuations increases with the increasing benefit that it brings. Respectively the time it takes to make a decision to fluctuate decreases with the increasing benefit that it brings. The fluctuation rate r1 introduced in the model partially takes into account fluctuation barriers of the second type mentioned here. Fluctuation barriers of the type mentioned first have not yet been taken into account in my model.

For the sake of completeness, it should be mentioned here that there are also phenomena that promote fluctuation. By changing jobs or professions, you acquire new skills and abilities and thus increase the value of your workforce. An occasional change also helps to exit a monotonous work routine and can thus cause a boost in motivation and more enjoyment at work. By various jobs you get to know your skills better and thus gain certainty whether you have made the right career choice or whether you should change again. I want to leave it open here whether and how these phenomena can or must be taken into account in future models.

9.7 Final remarks

The model described is applicable for explaining essential features of a labor market and showing which objective criteria should be used to differentiate wages in different professional groups in a meaningful way.

An interesting and not to be expected conclusion is that the principle of relative wage reductions (and increases) depending on supply and demand in the various professions in the labor market seems to be a functioning optimization strategy, while it is to be rejected as a price building principle in the goods market, because of the resulting instabilities (See Volume 1 **[4]**). However, some additional conditions must be taken into account, which in the capitalist market economy are not automatically given, if at all. In this way it must be prevented that necessary relative wage changes are misused to lower the general wage level.

A sheer performance wage is also not yet socially compatible. By introducing a minimum wage, however, it can be shaped socially without giving up the performance principle.

Overall, my investigations are far from conclusively addressing the problem. In the sub-items of section 9, some open questions have already been addressed.

Nevertheless, I have hereby ended my investigations for the time being, because a level has been reached with which I can go public in a relatively closed presentation in the hope of winning partners for further work on the problem. Further engagement with this matter would exceed my limited processing capacity.

Unfortunately, I cannot yet come up with an extensive literature analysis on this topic, because I have not yet found a similar starting point. I am grateful for references to other interesting literature sources on this problem, especially for those that also provide references to ways of quantifying meaningful income differentiations.

Appendix

List of symbols used [1] [3]

a	number of employees
ac	employment matrix
$ac_{j2,j3}$	number of employees of the productivity group j2 in profession j3. Component of matrix ac
ae_{j3}	number of all employees of profession j3
ap	vector of productivity group sizes.
ap_{j2}	number of workers belonging to productivity group j2.
d	vector of work performance
d_{j3}	total work performance of profession j3. Component of the vector d
db	vector of the needs-based share of work performance
db_{j3}	needs-based share of work performance d_{j3}. Component of the vector db
db0	vector of a needs-based performance structure
$db0_{j3}$	j3th component of the vector db0
dbn	vector of a normalized, needs-based performance structure
dbn_{j3}	j3th component of the vector dbn
fb_{j3}	share of needs-based work performance in the total work performance j3
fbg	factor of the needs-conformity of the work performance of society as a whole
fg	factor of overall societal increase in work performance through employment policy
fp_{j3}	factor of productivity increase by employment policy in profession j3
i4	index for numbering the reproductive cycle
j2	index for numbering the productivity groups
j3	index for numbering the professions
m2	number of productivity groups
m3	number of professions
Md	vector of the units of measure of job-specific work performance
Md_{j3}	unit of measure of job-specific work performance of profession j3. Component of the vector
Mp	normal for the price unit (e.g. Mp=1000 DM)
Mt	unit of time to which all work performances are referred to
l	matrix of currently realizable individual wages
$l_{j2,j3}$	currently realizable individual wage of a worker in productivity group j2 in profession j3
$ld_{j2,j3}$	current individual performance wage of a worker in full employment in productivity group j2 in profession j3
$lmax_{j2}$	current maximum realizable wage of a worker in productivity group j2
ls	current piece wage vector
ls_{j3}	current piece wage in profession j3. Component of the vector ls.
p	productivity matrix
p_{j2}	productivity profile of performance group j2. Row vector of the matrix p
$p_{j2,j3}$	productivity of a worker in productivity group j2 in profession j3. Component of the matrix p
pn	vector of normalized productivities
pn_{j3}	normalized productivity in profession j3 or average job-independent productivity in profession j3. Component of the vector pn
pm	vector of the mean profession-dependent productivities
pm_{j3}	Average productivity in profession j3 of those actually employed in this profession. Component of the vector pm
r1	fluctuation rate
r2	parameters of the speed of wage adjustment

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Table of figures

[1] Symbols used for the first time in Section 8.1 are not included in the symbol index because they are used exclusively in this section.

[2] This is not to say, of course, that unions and collective bargaining are superfluous under the current conditions of the capitalist market economy.

[3] The dimensional parameters are written starting with a capital letter and the corresponding dimensionless parameters start with a small letter. Therefore, the dimensionless parameters, for which there is a corresponding dimensional parameter, are not listed separately.